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XIV.

ON THE APPLICATION OF LOGICAL ANALYSIS TO
MULTIPLE ALGEBRA.

BY C. S. PEIRCE.

Presented, May 11, 1875.

THE letters of an algebra express the relation of the product to the multiplicand. Thus, iA expresses the quantity which is related to A in the manner denoted by i . This being the conception of these algebras, for each of them we may imagine another "absolute" algebra, as we may call it, which shall contain letters which can only be products and multiplicands, not multipliers. Let the general expression of the absolute algebra be $aI + bJ + cK + dL + \text{etc.}$ Multiply this by any letter i of the relative algebra, and denote the product by

$$\begin{aligned} & (A_1a + A_2b + A_3c + \text{etc.}) I \\ & + (B_1a + B_2b + B_3c + \text{etc.}) J \\ & + \text{etc.} \end{aligned}$$

Now we may obviously enlarge the given relative algebra, so that

$$\begin{aligned} i &= A_1 i_{11} + A_2 i_{12} + A_3 i_{13} + \text{etc.} \\ &+ B_1 i_{21} + B_2 i_{22} + B_3 i_{23} + \text{etc.} \\ &+ \text{etc.} \end{aligned}$$

where i_{11} i_{12} etc. are such, that the product of either of them into any letter of the absolute algebra shall equal some letter of that algebra. That there is no self-contradiction involved in this supposition seems axiomatic.

In this way each letter of the given algebra is resolved into a sum of terms of the form $aA : B$, a being a scalar, and $A : B$ such that

$$\begin{aligned} (A : B) (B : C) &= A : C. \\ (A : B) (C : D) &= 0. \end{aligned}$$

The actual resolution is usually performed with ease, but in some cases a good deal of ingenuity is required. I have not found the process facilitated by any general rules. I have actually resolved all the Double, Triple, and Quadruple algebras, and all the Quintuple ones, that appeared to present any difficulty. I give a few examples.

$$bi_3.$$

	i	j	k	l	m
i	j	0	l	0	0
j	0	0	0	0	0
k	$j + al$	0	0	0	$bj + cl$
l	0	0	0	0	0
m	$a'j + b'l$	0	$c'j + d'l$	0	l

$$i = cd' A : B + b' B : C + b' D : E.$$

$$j = b'cd' A : C.$$

$$k = cd' A : B + acd' D : B + b'c^2d' D : F + cd' E : C + bb'cd' A : F.$$

$$l = b'cd' D : C.$$

$$m = a'cd' A : B + b'c' A : E + b'cd' D : B + b'd' D : E + b'cd' D : F + F : C.$$

$$bd_3.$$

	i	j	k	l	m
i	j	0	l	0	0
j	0	0	0	0	0
k	$j + rl$	0	$i + m$	0	$-j - rl$
l	0	0	j	0	0
m	$(r^2 - 1)j$	0	$-l$	0	$-r^2j$

$$i = A : D + D : F + B : E + C : F.$$

$$j = A : F.$$

$$k = rA : B + rB : C + D : E - \frac{1}{r} D : F + E : F.$$

$$l = A : E - \frac{1}{r} A : F + B : F.$$

$$m = r^2 A : C - A : D - B : E - C : F.$$

$bh_6.$

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>
<i>i</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	
<i>j</i>	<i>j</i>	<i>k</i>				
<i>k</i>	<i>k</i>					
<i>l</i>	<i>l</i>	<i>ak</i>		<i>k</i>		
<i>m</i>						<i>k</i>
<i>n</i>	<i>n</i>					

$i = A:A + B:B + C:C + D:D.$
 $j = A:B + B:C.$
 $k = A:C.$
 $l = aA:B + A:D + D:C.$
 $m = A:E.$
 $n = E:C.$

$br_5.$

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>
<i>i</i>	<i>j</i>				
<i>j</i>					
<i>k</i>	<i>l</i>		<i>m</i>		
<i>l</i>					
<i>m</i>					

$i = A:B + B:C.$
 $j = A:C.$
 $k = D:B + D:E + E:F.$
 $l = D:C.$
 $m = D:F.$